CROSS

A signature scheme with restricted errors

Violetta Weger

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CodeMath 24

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CROSS - Timeline

2016 NIST standardization call for post-quantum PKE/KEM and signatures

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	1st round candidates:	40 submissions	11 code-based

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CROSS - Main Features

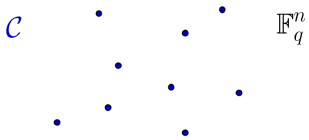
Implementation

- optimized AVX2
- memory-optimized
- constant worst-case runtime

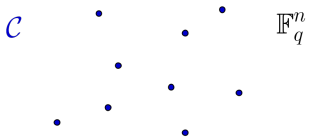
fast < 1 MCycle (NIST cat. I)
fits on Cortex-M4 microcontroller
no signature rejection

Ingredients

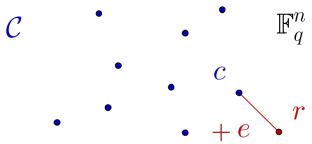
- Restricted Syndrome Decoding
  - → compact objects & efficient arithmetic
  - → NP-hard problem
- Zero-Knowledge protocol
  - → simple and well-studied
  - → EUF-CMA security
  - → BUFF security
  - → standard optimizations



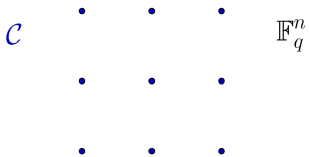
- Code $\mathcal{C} \subseteq \mathbb{F}_q^n$ linear subspace
- G generator matrix $\rightarrow$ $c = mG$



- Code $\mathcal{C} \subseteq \mathbb{F}_q^n$ linear subspace
- H parity-check matrix $\rightarrow$ $cH^\top = 0$

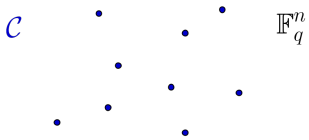


- Code $\mathcal{C} \subseteq \mathbb{F}_q^n$ linear subspace
- H parity-check matrix $\rightarrow$ $rH^\top = eH^\top = s$
- Hamming weight: $\mathrm{wt}(e) = |\{i \mid e_i \neq 0\}|$



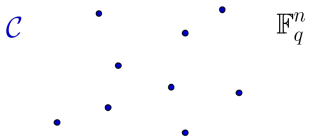
- algebraic structure
- e.g. RS, Goppa codes

$\rightarrow$ efficient decoders



- random code
- decoding is NP-hard

$\rightarrow$ Information set decoding



- random code
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$\rightarrow$ Information set decoding

Syndrome Decoding Problem (SDP)

Given p.c. matrix H, syndrome s, target weight t, find e s.t.

1. $s = eH^\top$ 2. $\mathrm{wt}(e) = t$

Signature Scheme

Signer

secret

Verifier

public

Signature Scheme

Signer

secret

Verifier

public

Signature Scheme

Signer

secret

Verifier

public

CROSS - Zero-Knowledge Protocol

ZK Protocol

Prover

secret

Interaction

Verifier

public

✓

CROSS - Zero-Knowledge Protocol

Signature Scheme

Signer

secret

Fiat-Shamir

Verifier

public

✓

Signature Scheme

Impersonator

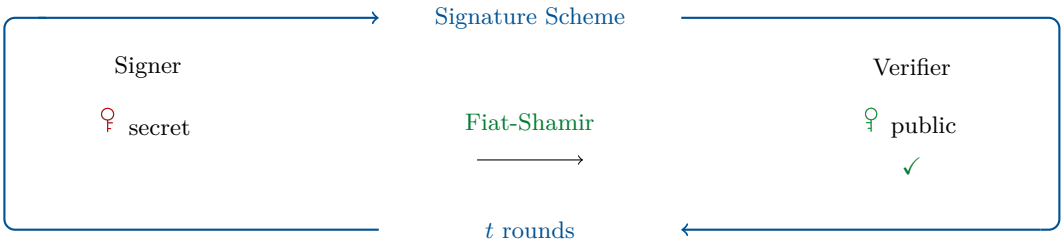
cheating prob.

Fiat-Shamir

Verifier

public

✓


Signature Scheme
Signer
secret
Fiat-Shamir
Verifier
public
✓
t rounds

CROSS - Zero-Knowledge Protocol

ZK Protocol

Prover

secret

Interaction

Verifier

public

✓

SDP Given H, s, t, find e s.t. 1. $s = eH^\top$, 2. $\mathrm{wt}(e) = t$

CROSS - Zero-Knowledge Protocol

ZK Protocol

Prover

secret

Interaction

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public

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e of $\mathrm{wt}(e) = t$

H, s, t

1. ✓ / 2. ✓

CROSS - Zero-Knowledge Protocol

ZK Protocol

Prover

secret

Interaction

Verifier

public

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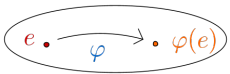

H, s, t

φ: 1. ✓ / $\varphi(e)$: 2. ✓

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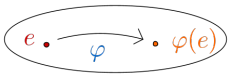

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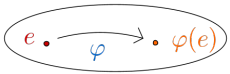

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1. Problem cheating prob. $\sim \frac{1}{2}$ → many rounds

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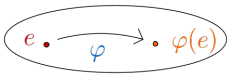

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1. Problem

→ Solution

cheating prob. $\sim \frac{1}{2}$

MPCitH: change protocol

→ many rounds

→ slow

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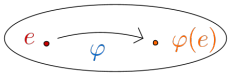

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$S = \{e \mid \mathrm{wt}(e) = t\}$	$\varphi : S \to S$ linear, transitive	→ $\lvert\varphi\rvert$ large

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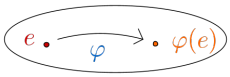

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$\rightarrow$ Solution	MPCitH: change protocol	$\rightarrow$ slow
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2. Problem	1 round: large commun. cost	

CROSS - Problems

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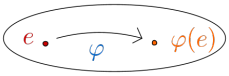

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→ Solution	change underlying problem	→ CROSS

Syndrome Decoding Problem: Given p.c. matrix H, syndrome s, weight t, find e s.t.

lin. constraint: 1. $s = eH^\top$ 2. $\mathrm{wt}(e) = t$ non-lin. constraint

Restricted SDP (R-SDP) Given p.c. matrix H, syndrome s, restriction $\mathbb{E}$, find e s.t.

lin. constraint 1. $s = eH^\top$ 2. $e \in \mathbb{E}^n$ non-lin. constraint

$\mathbb{E} = \{g^i \mid i \in \{1, \ldots, z\}\} < \mathbb{F}_q^\star$ $g \in \mathbb{F}_q^\star$ of prime order z

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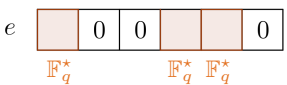
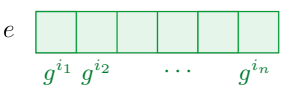


$\rightarrow$



- NP-hard
- adaption of ISD: exponential cost

Benefits

restriction $\mathbb{E} = \{g^i \mid i \in \{1, \ldots, z\}\}$

rest. vectors $e = (g^{i_1}, \ldots, g^{i_n}) \in \mathbb{F}_q^n$

Benefits

restriction $\mathbb{E} = \{g^i \mid i \in \{1, \ldots, z\}\}$ $\xrightarrow{\ell}$ exponents $\mathbb{F}_z^n$

rest. vectors $e = (g^{i_1}, \ldots, g^{i_n}) \in \mathbb{F}_q^n$ $\ell(e) = (i_1, \ldots, i_n)$

Benefits

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rest. vectors $e = (g^{i_1}, \ldots, g^{i_n}) \in \mathbb{F}_q^n$ $\quad \ell(e) = (i_1, \ldots, i_n)$

secret space $S = \mathbb{E}^n, \varphi : S \to S$ $\xrightarrow{\ell}$ $|e| = |\varphi| = n \log_2(z)$

$\varphi(e) = e' \star e, e' = (g^{j_1}, \ldots, g^{j_n})$

Benefits

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Example

$\mathbb{E} = \{1, 3, 9\} \subset \mathbb{F}_{13}$	$\xrightarrow{\ell}$	exponents in $\mathbb{F}_3^4$
$e = (1, 9, 3, 3)$		$\ell(e) = (0, 2, 1, 1)$
$\downarrow \ \star(3, 3, 9, 1)$		$\downarrow \ +(1, 1, 2, 0)$
$\tilde{e} = (3, 1, 1, 3)$		$\ell(\tilde{e}) = (1, 0, 0, 1)$

R-SDP Given H, s, $\mathbb{E}$, find e s.t. 1. $s = eH^\top$ 2. $e \in \mathbb{E}^n$ $(\mathbb{E}^n, \star) \simeq (\mathbb{F}_z^n, +)$

CROSS - R-SDP(G)

R-SDP(G) Given H, s, G, find e s.t. 1. $s = eH^\top$ 2. $e \in G$ $(G, \star) < (\mathbb{E}^n, \star)$

Benefits

$$x_1 = (g^{i_1}, \dots, g^{i_n})$$
$$\vdots$$
$$x_m = (g^{j_1}, \dots, g^{j_n})$$

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Benefits

$$\begin{matrix} x_1 = (g^{i_1}, \dots, g^{i_n}) \\ \vdots \\ x_m = (g^{j_1}, \dots, g^{j_n}) \end{matrix} \xrightarrow{\ell} M = \begin{pmatrix} i_1 & \cdots & i_n \\ \vdots & & \vdots \\ j_1 & \cdots & j_n \end{pmatrix} \in \mathbb{F}_z^{m \times n}$$

R-SDP(G) Given H, s, G, find e s.t. 1. $s = eH^\top$ 2. $e \in G$ $G \simeq \mathcal{C} \subset \mathbb{F}_z^n$

Benefits

$$x_1 = (g^{i_1}, \dots, g^{i_n}) \quad \vdots \quad x_m = (g^{j_1}, \dots, g^{j_n}) \xrightarrow{\ell} M = \begin{pmatrix} i_1 & \cdots & i_n \\ \vdots & & \vdots \\ j_1 & \cdots & j_n \end{pmatrix} \in \mathbb{F}_z^{m \times n}$$

$$e = {x_1}^{u_1} \star \cdots \star {x_m}^{u_m} \qquad \ell(e) = (u_1, \dots, u_m)M$$

$$\varphi : G \to G, \varphi(e) = e' \star e \xrightarrow{\ell} |e| = |\varphi| = m \log_2(z) < 1.5\lambda$$

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$\xrightarrow{\ell}$

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$\ell(e) = (u_1, \dots, u_m)M$

$\varphi : G \to G, \varphi(e) = e' \star e$

$\xrightarrow{\ell}$

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Example

$\mathbb{E} = \{1, 3, 9\} \subset \mathbb{F}_{13}$

$\xrightarrow{\ell}$

exponents in $\mathbb{F}_3^4$

$x_1 = (3, 1, 1, 3)$

$x_2 = (1, 3, 9, 1)$

$M = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 \end{pmatrix}$

$e = {x_1}^{②} \star {x_2}^{①} = (9, 3, 9, 9)$

$\ell(e) = (2, 1, 2, 2) = (2, 1)M$

- $\mathbb{E}, G$ have multiplicative structure
  $e = (g^{i_1}, \ldots, g^{i_n})$
- $s = eH^\top$ has additive structure
  $s_j = \sum_{\ell=1}^{n} h_{j,\ell} g^{i_\ell}$ for $j \in \{1, \ldots, n-k\}$

- $\mathbb{E}, G$ have multiplicative structure
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CROSS - Attacks

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- Take $\mathbb{E}$ with no additive structure
- good: $q = 13, g = 3, \mathbb{E} = \{1, 3, 9\}$
- bad: $q = 13, g = 5, \mathbb{E} = \{1, 5, -1, -5\}$

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- combinatorial:
  ISD algorithms

S. Bitzer, A. Pavoni, V. Weger, P. Santini, M. Baldi, and A. Wachter-Zeh. "Generic Decoding of Restricted Errors", ISIT, 2023.

M. Baldi, S. Bitzer, A. Pavoni, P. Santini, A. Wachter-Zeh, and V. Weger. "Zero knowledge protocols and signatures from the restricted syndrome decoding problem", PKC, 2024.

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- combinatorial:
  ISD algorithms
- algebraic attacks:
  $e_i^z = 1$ Gröbner basis

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M. Baldi, et al. "CROSS", NIST PQC round 1, 2023.

W. Beullens, P. Briaud, M. Øygarden. "A Security Analysis of Restricted Syndrome Decoding Problems", 2024.

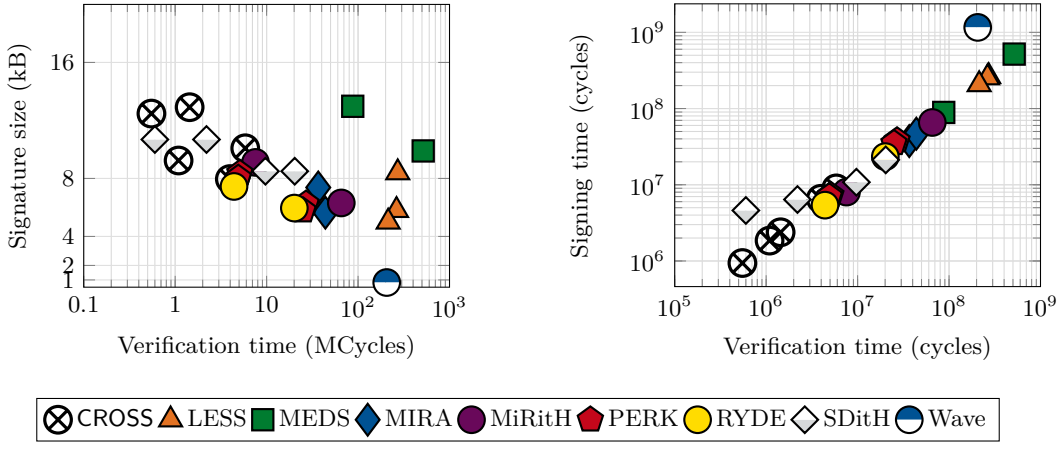
NIST cat. I

Problem	q, z	**Type**	(n, k, m)	**rounds**	**|Sign.|** (kB)	**Sign** (MCycles)	**Verify** (MCycles)
R-SDP	$(127, 7)$	fast	$(127, 76, -)$	163	19.1	1.28	0.78
		balanced		252	12.9	2.38	1.44
		short		960	10.1	8.96	5.84
R-SDP(G)	$(509, 127)$	fast	$(55, 36, 25)$	153	12.5	0.94	0.55
		balanced		243	9.2	1.85	1.09
		short		871	7.9	6.54	3.96

private and public keys < 0.1 kB key gen. < 0.1 MCycle

Measurements collected on an AMD Ryzen 5 Pro 3500U, clocked at 2.1GHz. The computer was running Debian GNU/Linux 12

CROSS - Comparison



Timings taken from `https://pqshield.github.io/nist-sigs-zoo/`

What's next?

- Hardware implementation
- Side-channel protection
- Worst-case to average-case reduction
- Smaller signatures?

Slides

CROSS

Codes & Restricted Objects Signature Scheme
`http://cross-crypto.com/`

Website

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Thank you!

CVE

PROVER		VERIFIER
KEY GENERATION		
Choose e with $\mathrm{wt}(e) = t$ H parity-check matrix Compute $s = eH^\top$	$\xrightarrow{\mathcal{P}=(H,s,t)}$	
		VERIFICATION
Choose $u \in \mathbb{F}_q^n$, $\varphi \in \mathcal{M}_n$ Set $c_1 = \mathrm{Hash}(\varphi, uH^\top)$ Set $c_2 = \mathrm{Hash}(\varphi(u), \varphi(e))$	$\xrightarrow{c_1,c_2}$	
	$\xleftarrow{z}$	Choose $z \in \mathbb{F}_q^\times$
Set $y = \varphi(u + ze)$	$\xrightarrow{y}$	
$r_1 = \varphi$	$\xleftarrow{b}$	Choose $b \in \{1, 2\}$
$r_2 = \varphi(e)$	$\xrightarrow{r_b}$	$b = 1$: $c_1 = \mathrm{Hash}(\varphi, \varphi^{-1}(y)H^\top - zs)$ $b = 2$: $\mathrm{wt}(\varphi(e)) = t$ and $c_2 = \mathrm{Hash}(y - z\varphi(e), \varphi(e))$

CVE

PROVER		VERIFIER
KEY GENERATION		Recall SDP: (1) $s = eH^\top$ (2) $\mathrm{wt}(e) = t$
Choose e with $\mathrm{wt}(e) = t$ H parity-check matrix Compute $s = eH^\top$	$\xrightarrow{\mathcal{P}=(H,s,t)}$	
		VERIFICATION
Choose $u \in \mathbb{F}_q^n$, $\varphi \in \mathcal{M}_n$ Set $c_1 = \mathrm{Hash}(\varphi, uH^\top)$ Set $c_2 = \mathrm{Hash}(\varphi(u), \varphi(e))$	$\xrightarrow{c_1, c_2}$	
	$\xleftarrow{z}$	Choose $z \in \mathbb{F}_q^\times$
Set $y = \varphi(u + ze)$	$\xrightarrow{y}$	
$r_1 = \varphi$	$\xleftarrow{b}$	Choose $b \in \{1, 2\}$
$r_2 = \varphi(e)$	$\xrightarrow{r_b}$	$b = 1$: $c_1 = \mathrm{Hash}(\varphi, \varphi^{-1}(y)H^\top - zs)$ $b = 2$: $\mathrm{wt}(\varphi(e)) = t$ and $c_2 = \mathrm{Hash}(y - z\varphi(e), \varphi(e))$

CVE

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KEY GENERATION		
Choose e with $\mathrm{wt}(e) = t$ H parity-check matrix Compute $s = eH^\top$	$\xrightarrow{\mathcal{P}=(H,s,t)}$	
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	$\xleftarrow{z}$	Choose $z \in \mathbb{F}_q^\times$
Set $y = \varphi(u + ze)$	$\xrightarrow{y}$	
$r_1 = \varphi$	$\xleftarrow{b}$	Choose $b \in \{1, 2\}$
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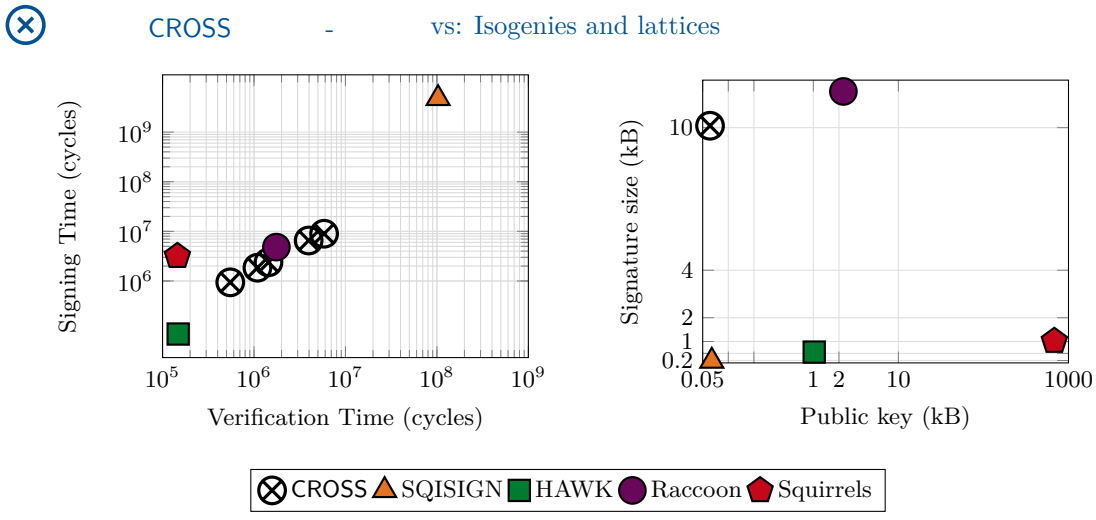
CROSS - vs: Isogenies and lattices
Signing Time (cycles)
10^9
10^8
10^7
10^6
10^5
10^6
10^7
10^8
10^9
Verification Time (cycles)
Signature size (kB)
10
4
2
1
0.2
0.05
1
2
10
1000
Public key (kB)
CROSS
SQISIGN
HAWK
Raccoon
Squirrels

CROSS - vs: Multivariate