CROSS

A signature scheme with restricted errors

Violetta Weger

CBCrypto 2025

May 4, 2025

Darmstadt, 2019

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A signature scheme with restricted errors

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CBCrypto 2025

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cbcrypto@googlegroups.com im Auftrag von Violetta Weger <violetta.weger@tum.de>

Fr 31.01.2025 19:19

cross

An: pqc-submissions@nist.gov <pqc-submissions@nist.gov>;

Cc: cbcrypto@googlegroups.com <cbcrypto@googlegroups.com>;

Dear NIST PQC team,

We are hereby submitting version 2.0 of CROSS for the round two.
Please find the complete submission package at:

https://crypto.deib.polimi.it/submission.zip

Please let us know if you received this email correctly and can open the zip.

Best wishes,

the CROSS team

--

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--

// Expanding secret key

$\mathbf{e}, \mathbf{e}_G, \mathbf{H}, \overline{\mathbf{M}} \leftarrow \mathsf{ExpandSK}(\mathsf{Seed}_{\mathsf{sk}})$: $\mathbf{e}, \mathbf{H} \leftarrow \mathsf{ExpandSK}(\mathsf{Seed}_{\mathsf{sk}})$

// Computing the commitments

$\mathsf{Seed} \xleftarrow{\$} \{0,1\}^{\lambda}, \quad \mathsf{Salt} \xleftarrow{\$} \{0,1\}^{2\lambda}$

$(\mathsf{Seed}[1], \ldots, \mathsf{Seed}[t]) \leftarrow \mathsf{SeedLeaves}(\mathsf{Seed}, \mathsf{Salt})$

// Compute $\mathbf{v}[i]$ such that $\mathbf{v}[i] \star \mathbf{e}'[i] = \mathbf{e}$

for i **from** 1 **to** t **do**

$\mathbf{e}'_G[i], \mathbf{u}'[i] \leftarrow \mathsf{CSPRNG}_{\mathbb{F}_2^m \times \mathbb{F}_p^n}(\mathsf{Seed}[i] \mid \mathsf{Salt} \mid i+c)$: $\mathbf{e}'[i], \mathbf{u}'[i] \leftarrow \mathsf{CSPRNG}_{\mathbb{F}_2^n \times \mathbb{F}_p^n}(\mathsf{Seed}[i] \mid \mathsf{Salt} \mid i+c)$

$\mathbf{v}_G[i] \leftarrow \mathbf{e}_G - \mathbf{e}'_G[i]$

$\mathbf{e}'[i] \leftarrow \mathbf{e}'_G[i]\overline{\mathbf{M}}$

$\mathbf{v}[i] \leftarrow \mathbf{e} - \mathbf{e}'[i]$

  for j **from** 1 **to** n **do**

    $\mathbf{v}[i]_j \leftarrow g^{\mathbf{v}[i]_j}$

  $\mathbf{u}[i] \leftarrow \mathbf{v}[i] \star \mathbf{u}'[i]$

  $\mathbf{s}'[i] \leftarrow \mathbf{u}[i]\mathbf{H}^{\top}$

$\mathsf{cmt}_0[i] \leftarrow \mathsf{Hash}(\mathbf{s}'[i] \mid \mathbf{v}_G[i] \mid \mathsf{Salt} \mid i+c)$: $\mathsf{cmt}_0[i] \leftarrow \mathsf{Hash}(\mathbf{s}'[i] \mid \mathbf{v}[i] \mid \mathsf{Salt} \mid i+c)$

  $\mathsf{cmt}_1[i] \leftarrow \mathsf{Hash}(\mathsf{Seed}[i] \mid \mathsf{Salt} \mid i+c)$

$\mathsf{digest}_{\mathsf{cmt}_0} \leftarrow \mathsf{TreeRoot}(\mathsf{cmt}_0[1], \cdots, \mathsf{cmt}_0[t])$

$\mathsf{digest}_{\mathsf{cmt}_1} \leftarrow \mathsf{Hash}(\mathsf{cmt}_1[1] \mid \cdots \mid \mathsf{cmt}_1[t])$

$\mathsf{digest}_{\mathsf{cmt}} \leftarrow \mathsf{Hash}(\mathsf{digest}_{\mathsf{cmt}_0} \mid \mathsf{digest}_{\mathsf{cmt}_1})$

// Computing first challenge

$\mathsf{digest}_{\mathsf{Msg}} \leftarrow \mathsf{Hash}(\mathsf{Msg})$

$\mathsf{digest}_{\mathsf{chall}_1} \leftarrow \mathsf{Hash}(\mathsf{digest}_{\mathsf{Msg}} \mid \mathsf{digest}_{\mathsf{cmt}} \mid \mathsf{Salt})$

$\mathsf{chall}_1 \leftarrow \mathsf{CSPRNG}_{(\mathbb{F}_p^*)^t}(\mathsf{digest}_{\mathsf{chall}_1} \mid t+c)$

// Computing first response

for i **from** 1 **to** t **do**

  for j **from** 1 **to** n **do**

    $\mathbf{e}'[i]_j \leftarrow g^{\mathbf{e}'[i]_j}$

  $\mathbf{y}[i] \leftarrow \mathbf{u}'[i] + \mathsf{chall}_1[i]\mathbf{e}'[i]$

// Computing second challenge

$\mathsf{digest}_{\mathsf{chall}_2} \leftarrow \mathsf{Hash}(\mathbf{y}[1] \mid \cdots \mid \mathbf{y}[t] \mid \mathsf{digest}_{\mathsf{chall}_1})$

$\mathsf{chall}_2 \leftarrow \mathsf{CSPRNG}_{\mathcal{B}_{(t,w)}}(\mathsf{digest}_{\mathsf{chall}_2} \mid t+c+1)$

// Computing second response

$\mathsf{Proof} \leftarrow \mathsf{TreeProof}(\mathsf{cmt}_0[1], \cdots, \mathsf{cmt}_0[t], \mathsf{chall}_2)$

$\mathsf{Path} \leftarrow \mathsf{SeedPath}(\mathsf{Seed}, \mathsf{Salt}, \mathsf{chall}_2)$

for i **from** 1 **to** t **do**

  if $\mathsf{chall}_2[i] = 0$ **then**

    $\mathsf{resp}[i]_0 \leftarrow (\mathbf{y}[i], \mathbf{v}_G[i])$: $\mathsf{resp}[i]_0 \leftarrow (\mathbf{y}[i], \mathbf{v}[i])$

    $\mathsf{resp}[i]_1 \leftarrow \mathsf{cmt}_1[i]$

// Assembling signature

$\mathsf{Sgn} \leftarrow (\mathsf{Salt}, \mathsf{digest}_{\mathsf{cmt}}, \mathsf{digest}_{\mathsf{chall}_2}, \mathsf{Path}, \mathsf{Proof}, \mathsf{resp})$

// Expanding secret key

1 $\vec{\mathbf{e}}, \vec{\mathbf{e}}_C, \mathbf{H}, \vec{\mathbf{M}} \leftarrow \mathsf{ExpandSK}(\mathsf{Seed}_{\mathsf{sk}})$ | $\vec{\mathbf{e}}, \mathbf{H} \leftarrow \mathsf{ExpandSK}(\mathsf{Seed}_{\mathsf{sk}})$

// Computing the commitments

2 $\mathsf{Seed} \xleftarrow{\$} \{0,1\}^{\lambda}, \quad \mathsf{Salt} \xleftarrow{\$} \{0,1\}^{2\lambda}$

3 $(\mathsf{Seed}[1], \ldots, \mathsf{Seed}[t]) \leftarrow \mathsf{SeedLeaves}(\mathsf{Seed}, \mathsf{Salt})$

// Compute $\mathbf{v}[i]$ such that $\mathbf{v}[i] * \mathbf{e}'[i] = \mathbf{e}$

4 **for** i **from** 1 **to** t **do**

$\vec{\mathbf{e}}'_C[i], \mathbf{u}'[i] \leftarrow \mathsf{CSPRNG}_{\mathbb{F}_p^n \times \mathbb{F}_p^*}(\mathsf{Seed}[i] \,|\, \mathsf{Salt} \,|\, i + c)$ | $\vec{\mathbf{e}}'[i], \mathbf{u}'[i] \leftarrow \mathsf{CSPRNG}_{\mathbb{F}_p^n \times \mathbb{F}_p^*}(\mathsf{Seed}[i] \,|\, \mathsf{Salt} \,|\, i + c)$

5

$\mathbf{v}_C[i] \leftarrow \mathbf{e}_C - \mathbf{e}'_C[i]$

$\mathbf{e}'[i] \leftarrow \vec{\mathbf{e}}'_C[i]\mathbf{M}$

$\mathbf{v}[i] \leftarrow$

$\mathsf{cmt}_1 \leftarrow \mathsf{Hash}(\mathsf{cmt}_1[1]$

$\mathsf{digest}_{\mathsf{cmt}} \leftarrow \mathsf{Hash}(\mathsf{digest}_{\mathsf{cmt}_0}$ | $\mathsf{cmt}_0[i] \leftarrow \mathsf{Hash}(\mathbf{s}'[i] \,|\, \mathbf{v}[i] \,|\, \mathsf{Salt} \,|\, i + c)$

// Computing first challenge

$\mathsf{digest}_{\mathsf{Msg}} \leftarrow \mathsf{Hash}(\mathsf{Msg})$

$\mathsf{digest}_{\mathsf{chall}_1} \leftarrow \mathsf{Hash}(\mathsf{digest}_{\mathsf{Msg}} \mid \mathsf{d}$

$\mathsf{chall}_1 \leftarrow \mathsf{CSPRNG}_{(\mathbb{F}_p^*)^t}(\mathsf{digest}_{\mathsf{ch}}$

// Computing first response

for i **from** 1 **to** t **do**

for j **from** 1 **to** n **do**

$\mathbf{e}'[i]_j \leftarrow g^{\vec{\mathbf{e}}'[i]_j}$

$[i] \leftarrow \mathbf{u}'[i] + \mathsf{chall}_1[i]\mathbf{e}'[i]$

ting second cha

27 $\mathsf{resp}[i]_0 \leftarrow (\mathbf{y}[i], \mathbf{v}[i])$

$\mathsf{resp}[i]_1 \leftarrow \mathsf{cmt}_1[i]$

// Assembling signature

28 $\mathsf{Sgn} \leftarrow (\mathsf{Salt}, \mathsf{digest}_{\mathsf{cmt}}, \mathsf{digest}_{\mathsf{chall}_2}, \mathsf{Path}, \mathsf{Proof}, \mathsf{resp})$

CROSS - in a nutshell

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- code-based signature scheme
- 2nd round candidate in NIST standardization call
- Zero-Knowledge (ZK) protocol + Fiat-Shamir transform
- well-known ZK protocol based on decoding random code
- tweak: decoding restricted errors
- standard optimization techniques

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Advertisement

- optimized AVX2 — fast < 1 MCycle (NIST cat. I)
- memory-optimized — fits on Cortex-M4 microcontroller
- constant worst-case runtime — no signature rejection
- available on liboqs — and SUPERCOP
- EUF-CMA security — also BUFF

CROSS - Who is behind CROSS?

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Felice Manganiello


Freeman Slaughter


Antonia Wachter-Zeh


Sebastian Bitzer


Violetta Weger


Alessio Pavoni


Patrick Karl


Jonas Schupp


Marco Gianvecchio


Alessandro Barenghi


Michele Battagliola


Edoardo Signorini


Gerardo Pelosi


Paolo Santini


Marco Baldi

CROSS - Why “CROSS”?

Codes and Restricted Objects Signature Scheme

CROSS - ~~Why "CROSS"?~~ Why $\otimes$?

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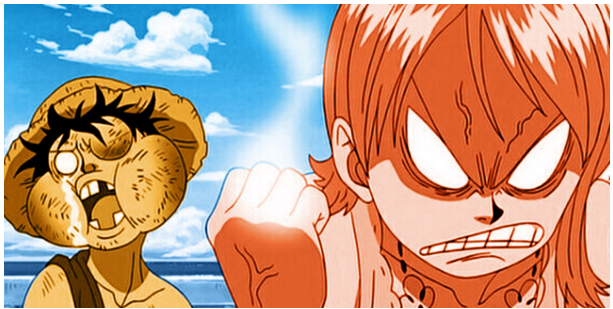

CROSS - Zero-Knowledge Protocol

CROSS - Zero-Knowledge Protocol

Prover

secret key

secret space

Verifier

public key

Prover

secret key

secret space

commitment c

Verifier

public key

Prover

secret key

secret space

commitment c

Verifier

public key

challenge z

CROSS - Zero-Knowledge Protocol

Prover

secret key

secret space

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commitment c

challenge z

response r

CROSS - Zero-Knowledge Protocol

Prover

secret key

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commitment c

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Verifier

public key

challenge z

verify , z, $r \rightarrow c$

Prover

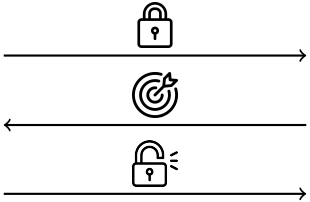
?

secret space

Verifier

public key

commitment c



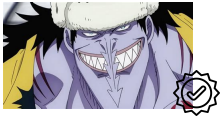
challenge z

response r

verify , z, $r \rightarrow c$

CROSS - Zero-Knowledge Protocol

Prover



small cheating probability

Verifier

public key

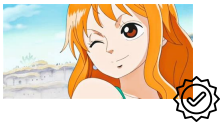
commitment c

challenge z

response r

verify , z, $r \rightarrow c$

Prover



Verifier

sec. level λ

cheat. prob. α

$\rightarrow t$ parallel rounds s.t. $\alpha^t < 2^{-\lambda}$

secret key

public key

commitment c

challenge z

response r

verify , z, $r \rightarrow c$

t rounds

CROSS - Zero-Knowledge Protocol

Signer

Fiat-Shamir transform

Verifier

secret key

m message

public key

commitment c

$z = \mathsf{Hash}(c, m)$

response r

$z = \mathsf{Hash}(c, m)$

verify , z, $r \rightarrow c$

$\rightarrow$ signature $= (c_i, r_i)_i$: transcript of all t rounds

Prover

♀

♡

SDP: Given H, s, t find e

1. $s = eH^\top$ 2. $\mathrm{wt}(e) = t$

Verifier

♀

Prover

$\mathrm{wt}(x) = t$

e

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Verifier

H, s, t

Prover

$\mathrm{wt}(x) = t$

φ e

$u \in \mathbb{F}_q^n, \varphi$

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$c_1 = \mathsf{Hash}(uH^\top, \varphi)$

$c_2 = \mathsf{Hash}(\varphi(u), \varphi(e))$

Verifier

H, s, t

Prover

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$y = \varphi(u + ze)$

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$r_1 = \varphi$

$r_2 = \varphi(e)$

Prover

$\mathrm{wt}(x) = t$

φ $\quad$ e

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$r_1 = \varphi$

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$\varphi^{-1}(y) = u + ze$

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$\varphi^{-1}(y)H^\top = uH^\top + zs$

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$y - z\varphi(e) = \varphi(u)$

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probability $\sim 1/2 \to t \sim \lambda$

H, s, t

$u \in \mathbb{F}_q^n, \varphi$

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CROSS - Why not submitted?

well-studied ☺

J. Stern. "A new identification scheme based on syndrome decoding", Annual Int. Cryptology Conf., 1993.

P.-L. Cayrel, P. Véron, S. El Yousfi Alaoui. "A zero-knowledge identification scheme based on the q-ary syndrome decoding problem", Int. Workshop on Selected Areas in Cryptography, 2011.

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too large ☹

large cheating probability $\rightarrow$ many rounds $\rightarrow$ large signatures ~ 43 kB for $\lambda = 128$

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solution

change protocol: use MPC to split error $\rightarrow$ smaller cheating probability

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large cheating probability → many rounds → large signatures ∼ 43 kB for $\lambda = 128$

solution

change protocol: use MPC to split error → smaller cheating probability

→ used in

- SDitH (regular)
- RYDE (rank)
- Mirath (matrix rank)
- PERK (PKP)

Prover

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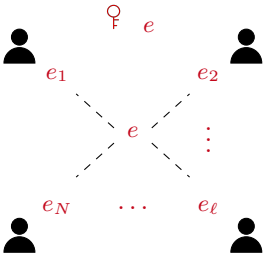
Verifier

H, s, t



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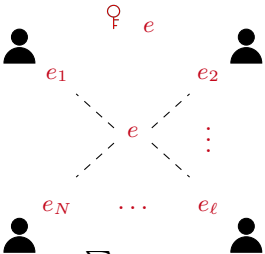
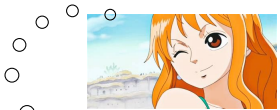
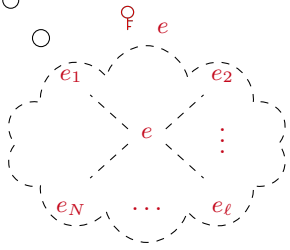


$$\sum e_i = e$$

1. $s_i = e_i H^\top \rightarrow \sum s_i = s$
2. $\alpha_i \rightarrow \mathrm{wt}(e) = t$

Prover





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Verifier

H, s, t

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SDP: Given H, s, t find e

1. $s = eH^\top$ 2. $\mathrm{wt}(e) = t$

♀ e

e_1 e_2 e e_N $\cdots$ e_ℓ

$\sum e_i = e$

1. $s_i = e_i H^\top \rightarrow \sum s_i = s$
2. $\alpha_i \rightarrow \mathrm{wt}(e) = t$

$c_i = (s_i, \alpha_i)$ →

♀ H, s, t

Prover

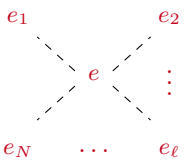
Verifier

SDP: Given H, s, t find e

1. $s = eH^\top$ 2. $\mathrm{wt}(e) = t$

P e

V H, s, t



$c_i = (s_i, \alpha_i)$

$z \in \{1, \ldots, N\}$

$$\sum e_i = e$$

1. $s_i = e_i H^\top \rightarrow \sum s_i = s$
2. $\alpha_i \rightarrow \mathrm{wt}(e) = t$

Prover

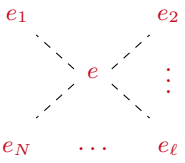
Verifier

SDP: Given H, s, t find e

1. $s = eH^\top$ 2. $\mathrm{wt}(e) = t$

H, s, t

e



$c_i = (s_i, \alpha_i)$

$z \in \{1, \ldots, N\}$

$r_z = (e_i)_{i \neq z}$

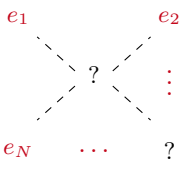
check $(c_i)_{i \neq z}$

$$\sum e_i = e$$

1. $s_i = e_i H^\top \rightarrow \sum s_i = s$
2. $\alpha_i \rightarrow \mathrm{wt}(e) = t$

Prover

???



$$\sum e_i = e$$

1. $s_i = e_i H^\top \rightarrow \sum s_i = s$
2. $\alpha_i \rightarrow \mathrm{wt}(e) = t$

SDP: Given H, s, t find e

1. $s = eH^\top$ 2. $\mathrm{wt}(e) = t$

$c_i = (s_i, \alpha_i)$

$z \in \{1, \ldots, N\}$

$r_z = (e_i)_{i \neq z}$

check $(c_i)_{i \neq z}$

Verifier

H, s, t

Prover

Verifier

SDP: Given H, s, t find e

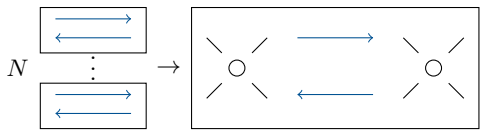
1. $s = eH^\top$ 2. $\mathrm{wt}(e) = t$

cheating probability $\sim 1/N$

e_1 e_2 ? e_N $\cdots$?

$c_i = (s_i, \alpha_i)$

$r_z = (e_i)_{i \neq z}$



H, s, t

$z \in \{1, \ldots, N\}$

check $(c_i)_{i \neq z}$

☺ smaller cheating prob → smaller signatures

Prover

Verifier

SDP: Given H, s, t find e

1. $s = eH^\top$ 2. $\mathrm{wt}(e) = t$

cheating probability $\sim 1/N$

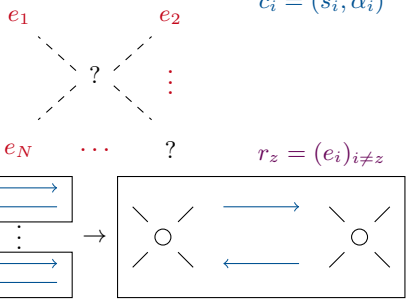
H, s, t

$c_i = (s_i, \alpha_i)$

$z \in \{1, \ldots, N\}$

check $(c_i)_{i \neq z}$

$r_z = (e_i)_{i \neq z}$



☺ smaller cheating prob → smaller signatures

☹ more computations → slow

CROSS - Why not submitted?

well-studied ☺

J. Stern. "A new identification scheme based on syndrome decoding", Annual Int. Cryptology Conf., 1993.

P.-L. Cayrel, P. Véron, S. El Yousfi Alaoui. "A zero-knowledge identification scheme based on the q-ary syndrome decoding problem", Int. Workshop on Selected Areas in Cryptography, 2011.

too large ☹

large cheating probability $\rightarrow$ many rounds $\rightarrow$ large signatures ~ 43 kB for $\lambda = 128$

CROSS - Why not submitted?

well-studied ☺

J. Stern. "A new identification scheme based on syndrome decoding", Annual Int. Cryptology Conf., 1993.

P.-L. Cayrel, P. Véron, S. El Yousfi Alaoui. "A zero-knowledge identification scheme based on the q-ary syndrome decoding problem", Int. Workshop on Selected Areas in Cryptography, 2011.

too large ☹

large communication cost in 1 round

CROSS - Why not submitted?

well-studied ☺

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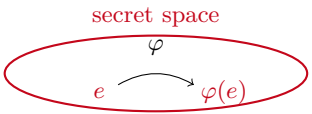
P.-L. Cayrel, P. Véron, S. El Yousfi Alaoui. "A zero-knowledge identification scheme based on the q-ary syndrome decoding problem", Int. Workshop on Selected Areas in Cryptography, 2011.

too large ☹

large communication cost in 1 round



errors $|e| = t(\log(n) + \log(q-1))$

maps $|\varphi| = n(\log(n) + \log(q-1))$

CROSS - Why not submitted?

well-studied ☺

J. Stern. "A new identification scheme based on syndrome decoding", Annual Int. Cryptology Conf., 1993.

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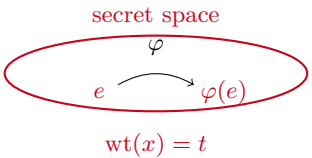
too large ☹

large communication cost in 1 round

solution

change problem: use restricted errors

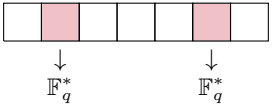


errors $|e| = t(\log(n) + \log(q-1))$

maps $|\varphi| = n(\log(n) + \log(q-1))$

SDP: Given H, s, t find e: 1. $s = eH^\top$, 2. $\mathrm{wt}(e) = t$

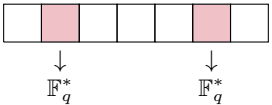
e



$\varphi \in (\mathbb{F}_q^*)^n \rtimes S_n$

SDP: Given H, s, t find e: 1. $s = eH^\top$, 2. $\mathrm{wt}(e) = t$

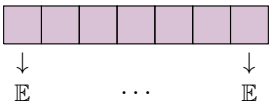
e



$\varphi \in (\mathbb{F}_q^*)^n \rtimes S_n$

R-SDP: Given $H, s, \mathbb{E}$ find e: 1. $s = eH^\top$, 2. $e \in \mathbb{E}^n$

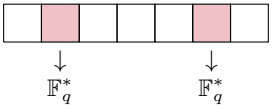
e



$\varphi : \mathbb{E}^n \to \mathbb{E}^n$?

SDP: Given H, s, t find e: 1. $s = eH^\top$, 2. $\mathrm{wt}(e) = t$

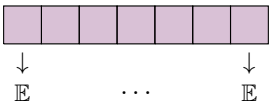
e



$\varphi \in (\mathbb{F}_q^*)^n \rtimes S_n$

R-SDP: Given $H, s, \mathbb{E}$ find e: 1. $s = eH^\top$, 2. $e \in \mathbb{E}^n$

e



$\varphi : \mathbb{E}^n \to \mathbb{E}^n$?

$\mathbb{E} < \mathbb{F}_q^*$ $\quad \mathbb{E} = \{g^i \mid i \in \{1, \ldots, z\}\}$ $\quad \mathrm{ord}(g) = z$ prime

R-SDP: Given $H, s, \mathbb{E}$ find e: 1. $s = eH^\top$, 2. $e \in \mathbb{E}^n$

$\mathbb{E}^n = \{g^i\}^n \subset \mathbb{F}_q^n$

e | g^{i_1} | $\cdots$ | g^{i_n} |

R-SDP: Given $H, s, \mathbb{E}$ find e: 1. $s = eH^\top$, 2. $e \in \mathbb{E}^n$

$\mathbb{E}^n = \{g^i\}^n \subset \mathbb{F}_q^n$

e | g^{i_1} | $\cdots$ | g^{i_n} |

↓

$\ell(e)$ | i_1 | $\cdots$ | i_n |

$\mathbb{F}_z^n$

R-SDP: Given $H, s, \mathbb{E}$ find e: 1. $s = eH^\top$, 2. $e \in \mathbb{E}^n$

$\mathbb{E}^n = \{g^i\}^n \subset \mathbb{F}_q^n$

e | g^{i_1} | $\cdots$ | g^{i_n} |

↓

$\ell(e)$ | i_1 | $\cdots$ | i_n |

$\mathbb{F}_z^n$

$\rightarrow |e| = n \log(z)$

R-SDP: Given $H, s, \mathbb{E}$ find e: 1. $s = eH^\top$, 2. $e \in \mathbb{E}^n$

$\mathbb{E}^n = \{g^i\}^n \subset \mathbb{F}_q^n$

e | g^{i_1} | $\cdots$ | g^{i_n} $\xrightarrow{\varphi}$ e' | g^{j_1} | $\cdots$ | g^{j_n}

$\downarrow$

$\ell(e)$ | i_1 | $\cdots$ | i_n

$\mathbb{F}_z^n$

$\rightarrow |e| = n \log(z)$

R-SDP: Given $H, s, \mathbb{E}$ find e: 1. $s = eH^\top$, 2. $e \in \mathbb{E}^n$

$\mathbb{E}^n = \{g^i\}^n \subset \mathbb{F}_q^n$

e | g^{i_1} | $\cdots$ | g^{i_n} $\star$ | $g^{j_1 - i_1}$ | $\cdots$ | $g^{j_n - i_n}$ e' | g^{j_1} | $\cdots$ | g^{j_n}

$\downarrow$

$\ell(e)$ | i_1 | $\cdots$ | i_n

$\mathbb{F}_z^n$

$\rightarrow |e| = n \log(z)$

R-SDP: Given $H, s, \mathbb{E}$ find e: 1. $s = eH^\top$, 2. $e \in \mathbb{E}^n$

$\mathbb{E}^n = \{g^i\}^n \subset \mathbb{F}_q^n$

e | g^{i_1} | $\cdots$ | g^{i_n} | $\star$ | $g^{j_1-i_1}$ | $\cdots$ | $g^{j_n-i_n}$ | e' | g^{j_1} | $\cdots$ | g^{j_n}

$\downarrow$

$\ell(e)$ | i_1 | $\cdots$ | i_n | $j_1 - i_1$ | $\cdots$ | $j_n - i_n$ | $\ell(e')$ | j_1 | $\cdots$ | j_n

$\mathbb{F}_z^n$

$\rightarrow |e| = n \log(z)$ $\rightarrow |\varphi| = n \log(z)$

R-SDP: Given $H, s, \mathbb{E}$ find e: 1. $s = eH^\top$, 2. $e \in \mathbb{E}^n$

$\mathbb{E}^n = \{g^i\}^n \subset \mathbb{F}_q^n$

e | g^{i_1} | $\cdots$ | g^{i_n} | $\star$ | $g^{j_1-i_1}$ | $\cdots$ | $g^{j_n-i_n}$ | e' | g^{j_1} | $\cdots$ | g^{j_n}

$\downarrow$

$\ell(e)$ | i_1 | $\cdots$ | i_n | $+$ | $j_1 - i_1$ | $\cdots$ | $j_n - i_n$ | $\ell(e')$ | j_1 | $\cdots$ | j_n

$\mathbb{F}_z^n$

$\rightarrow |e| = n \log(z)$ $\rightarrow |\varphi| = n \log(z)$

R-SDP: Given $H, s, \mathbb{E}$ find e: 1. $s = eH^\top$, 2. $e \in \mathbb{E}^n$

$\mathbb{E}^n = \{g^i\}^n \subset \mathbb{F}_q^n$

e | g^{i_1} | $\cdots$ | g^{i_n} | $\star$ | $g^{j_1-i_1}$ | $\cdots$ | $g^{j_n-i_n}$ | e' | g^{j_1} | $\cdots$ | g^{j_n}

$\downarrow$

$\ell(e)$ | i_1 | $\cdots$ | i_n | $+$ | $j_1 - i_1$ | $\cdots$ | $j_n - i_n$ | $\ell(e')$ | j_1 | $\cdots$ | j_n

$\mathbb{F}_z^n$

$\rightarrow |e| = n\log(z)$ $\rightarrow |\varphi| = n\log(z)$ $\rightarrow (\mathbb{E}^n, \star) \cong (\mathbb{F}_z^n, +)$

R-SDP: Given $H, s, \mathbb{E}$ find e: 1. $s = eH^\top$, 2. $e \in \mathbb{E}^n$

$\mathbb{E}^n = \{g^i\}^n \subset \mathbb{F}_q^n$

e | g^{i_1} | $\cdots$ | g^{i_n} | $\star$ | $g^{j_1-i_1}$ | $\cdots$ | $g^{j_n-i_n}$ | e' | g^{j_1} | $\cdots$ | g^{j_n}

$\downarrow$

$\ell(e)$ | i_1 | $\cdots$ | i_n | $+$ | j_1-i_1 | $\cdots$ | j_n-i_n | $\ell(e')$ | j_1 | $\cdots$ | j_n

$\mathbb{F}_z^n$

$\rightarrow |e| = n\log(z)$ $\rightarrow |\varphi| = n\log(z)$ $\rightarrow (\mathbb{E}^n, \star) \cong (\mathbb{F}_z^n, +)$

☺ smaller sizes ☺ faster arithmetic

$\mathbb{E} = \{3^0, 3^1, 3^2\} = \{1, 3, 9\} < \mathbb{F}_{13}$

$e = (1, 9, 3, 3)$ $e' = (3, 1, 1, 3)$

$\mathbb{E} = \{3^0, 3^1, 3^2\} = \{1, 3, 9\} < \mathbb{F}_{13}$

$e = (1, 9, 3, 3)$ $\star(3, 3, 9, 1)$ $e' = (3, 1, 1, 3)$

$\mathbb{E} = \{3^0, 3^1, 3^2\} = \{1, 3, 9\} < \mathbb{F}_{13}$

$e = (1, 9, 3, 3)$ $\star(3, 3, 9, 1)$ $e' = (3, 1, 1, 3)$

$\ell(e) = (0, 2, 1, 1)$ $+(1, 1, 2, 0)$ $\ell(e') = (1, 0, 0, 1)$

$(\mathbb{F}_3^n, +)$

$\mathbb{E} = \{3^0, 3^1, 3^2\} = \{1, 3, 9\} < \mathbb{F}_{13}$

$e = (1, 9, 3, 3)$ $\star(3, 3, 9, 1)$ $e' = (3, 1, 1, 3)$

$\downarrow$

$\ell(e) = (0, 2, 1, 1)$ $+(1, 1, 2, 0)$ $\ell(e') = (1, 0, 0, 1)$

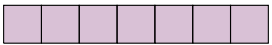
$(\mathbb{F}_3^n, +)$

allowing any exponent in $\mathbb{F}_z^n$ $\rightarrow$ only allow exponents in $\mathcal{C} \subset \mathbb{F}_z^n$

SDP: Given H, s, t find e: 1. $s = eH^\top$, 2. $\mathrm{wt}(e) = t$

e  $\in \mathbb{F}_q^*$

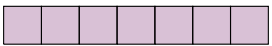
R-SDP: Given $H, s, \mathbb{E}$ find e: 1. $s = eH^\top$, 2. $e \in \mathbb{E}^n$

e  $\in \mathbb{E} < \mathbb{F}_q^*$ $\ell(\mathbb{E}^n) \cong \mathbb{F}_z^n$

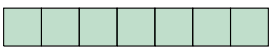
SDP: Given H, s, t find e: 1. $s = eH^\top$, 2. $\mathrm{wt}(e) = t$

e  $\in \mathbb{F}_q^*$

R-SDP: Given $H, s, \mathbb{E}$ find e: 1. $s = eH^\top$, 2. $e \in \mathbb{E}^n$

e  $\in \mathbb{E} < \mathbb{F}_q^*$ $\ell(\mathbb{E}^n) \cong \mathbb{F}_z^n$

R-SDP(G): Given $H, s, G < \mathbb{E}^n$ find e: 1. $s = eH^\top$, 2. $e \in G$

e  $\in G < \mathbb{E}^n$ $\ell(G) \cong \mathcal{C} \subset \mathbb{F}_z^n$

CROSS - What are restricted errors?

R-SDP(G): Given $H, s, G < \mathbb{E}^n$ find e: 1. $s = eH^\top$, 2. $e \in G$

$G \subset \mathbb{F}_q^n$

$(G, \star) < (\mathbb{E}^n, \star)$ $\quad G = \langle x_1, \ldots, x_m \rangle_\star$ $\quad e = x_1^{u_1} \star \cdots \star x_m^{u_m}$

R-SDP(G): Given $H, s, G < \mathbb{E}^n$ find e: 1. $s = eH^\top$, 2. $e \in G$

$G \subset \mathbb{F}_q^n$

$(G, \star) < (\mathbb{E}^n, \star)$ $\quad G = \langle x_1, \ldots, x_m \rangle_\star$ $\quad e = x_1^{u_1} \star \cdots \star x_m^{u_m}$

$\downarrow$

$$M = \begin{pmatrix} \ell(x_1) \\ \vdots \\ \ell(x_m) \end{pmatrix} \in \mathbb{F}_z^{m \times n} \qquad \ell(e) = uM \in \langle M \rangle \qquad \rightarrow \text{only need } u \in \mathbb{F}_z^m$$

$\mathbb{F}_z^n$

R-SDP(G): Given $H, s, G < \mathbb{E}^n$ find e: 1. $s = eH^\top$, 2. $e \in G$

$G \subset \mathbb{F}_q^n$

$(G, \star) < (\mathbb{E}^n, \star)$ $\quad G = \langle x_1, \ldots, x_m \rangle_\star$ $\quad e = x_1^{u_1} \star \cdots \star x_m^{u_m}$

$\downarrow$

$$M = \begin{pmatrix} \ell(x_1) \\ \vdots \\ \ell(x_m) \end{pmatrix} \in \mathbb{F}_z^{m \times n} \qquad \ell(e) = uM \in \langle M \rangle \qquad \rightarrow \text{only need } u \in \mathbb{F}_z^m$$

$\mathbb{F}_z^n$

$$\underset{u = (u_1, \ldots, u_m)}{\mathbb{F}_z^m} \xrightarrow{M} \underset{\ell(e) = uM = (i_1, \ldots, i_n)}{\mathbb{F}_z^n} \xrightarrow{g} \underset{e = (g^{i_1}, \ldots, g^{i_n})}{\mathbb{E}^n \subset \mathbb{F}_q^n}$$

R-SDP(G): Given $H, s, G < \mathbb{E}^n$ find e: 1. $s = eH^\top$, 2. $e \in G$

$G \subset \mathbb{F}_q^n$

$(G, \star) < (\mathbb{E}^n, \star)$ $\quad G = \langle x_1, \ldots, x_m \rangle_\star$ $\quad e = x_1^{u_1} \star \cdots \star x_m^{u_m}$

$\downarrow$

$$M = \begin{pmatrix} \ell(x_1) \\ \vdots \\ \ell(x_m) \end{pmatrix} \in \mathbb{F}_z^{m \times n} \qquad \ell(e) = uM \in \langle M \rangle \qquad \rightarrow \text{only need } u \in \mathbb{F}_z^m$$

$\mathbb{F}_z^n$

$\rightarrow |e| = |\varphi| = m \log(z)$ $\quad$ ☺ even smaller

$G < \mathbb{E}^n = \{1, 3, 9\}^n \subset \mathbb{F}_{13}^n$

$G = \langle x_1, x_2 \rangle$ $x_1 = (3, 1, 1, 3), x_2 = (1, 3, 9, 1)$ $e = {x_1}^2 \star {x_2}^1 = (9, 3, 9, 9)$

$G < \mathbb{E}^n = \{1, 3, 9\}^n \subset \mathbb{F}_{13}^n$

$G = \langle x_1, x_2 \rangle \qquad x_1 = (3, 1, 1, 3), x_2 = (1, 3, 9, 1) \qquad e = {x_1}^2 \star {x_2}^1 = (9, 3, 9, 9)$

$\downarrow$

$\ell(G) = \langle M \rangle \subset \mathbb{F}_3^n \qquad M = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 \end{pmatrix} \qquad \ell(e) = (2, 1)M = (2, 1, 1, 2)$

$\langle M \rangle \subset \mathbb{F}_3^n$

$G < \mathbb{E}^n = \{1, 3, 9\}^n \subset \mathbb{F}_{13}^n$

$G = \langle x_1, x_2 \rangle$ $\quad x_1 = (3, 1, 1, 3), x_2 = (1, 3, 9, 1)$ $\quad e = {x_1}^2 \star {x_2}^1 = (9, 3, 9, 9)$

$\downarrow$

$\ell(G) = \langle M \rangle \subset \mathbb{F}_3^n$ $\quad M = \begin{pmatrix} 1 & 0 & 0 & 1 \\ 0 & 1 & 2 & 0 \end{pmatrix}$ $\quad \ell(e) = (2, 1)M = (2, 1, 1, 2)$

$\langle M \rangle \subset \mathbb{F}_3^n$

previous $e = (1, 9, 3, 3) \notin G$

- $\mathbb{E}, G$ have multiplicative structure
  $e = (g^{i_1}, \ldots, g^{i_n})$
- $s = eH^\top$ has additive structure
  $s_j = \sum_{\ell=1}^{n} h_{j,\ell} g^{i_\ell}$ for $j \in \{1, \ldots, n-k\}$

- $\mathbb{E}, G$ have multiplicative structure
  $e = (g^{i_1}, \ldots, g^{i_n})$ not DLP
- $s = eH^\top$ has additive structure
  $s_j = \sum_{\ell=1}^{n} h_{j,\ell} g^{i_\ell}$ for $j \in \{1, \ldots, n-k\}$

- $\mathbb{E}, G$ have multiplicative structure
  $e = (g^{i_1}, \ldots, g^{i_n})$ not DLP
- similar to SDP $\rightarrow$ NP-hard
- $s = eH^\top$ has additive structure
  $s_j = \sum_{\ell=1}^{n} h_{j,\ell} g^{i_\ell}$ for $j \in \{1, \ldots, n-k\}$
- adapted ISD algorithms

- $\mathbb{E}, G$ have multiplicative structure
  $e = (g^{i_1}, \ldots, g^{i_n})$ not DLP
- similar to SDP $\rightarrow$ NP-hard
- generalize to $e \in (\mathbb{E} \cup \{0\})^n$, $\mathrm{wt}(e) = t$
- choose t s.t. unique solution ($q = 251$)

- $s = eH^\top$ has additive structure
  $s_j = \sum_{\ell=1}^{n} h_{j,\ell} g^{i_\ell}$ for $j \in \{1, \ldots, n-k\}$
- adapted ISD algorithms

- $\mathbb{E}, G$ have multiplicative structure
  $e = (g^{i_1}, \ldots, g^{i_n})$ not DLP
- similar to SDP $\rightarrow$ NP-hard
- generalize to $e \in (\mathbb{E} \cup \{0\})^n$, $\mathrm{wt}(e) = t$
- choose t s.t. unique solution ($q = 251$)

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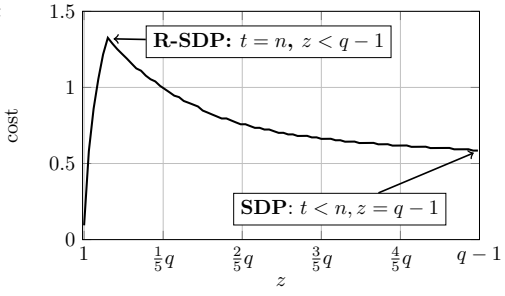

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- $(\mathbb{E} + \mathbb{E}) \cap (\mathbb{E} \cup \{0\})$ small

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  - good: $q = 13, g = 3, \mathbb{E} = \{1, 3, 9\}$
  - bad: $q = 13, g = 5, \mathbb{E} = \{\pm 1, \pm 5\}$

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  $\rightarrow \quad q = 127, z = 7$

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S. Bitzer, A. Pavoni, V. Weger, P. Santini, M. Baldi, and A. Wachter-Zeh. "Generic Decoding of Restricted Errors", ISIT, 2023.

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M. Baldi, et al. "CROSS security guide", NIST PQC round 2, 2025.

W. Beullens, P. Briaud, M. Øygarden. "A Security Analysis of Restricted Syndrome Decoding Problems", 2024.

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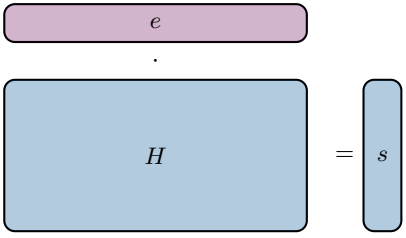
1. Reduction



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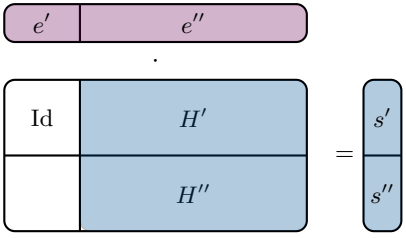

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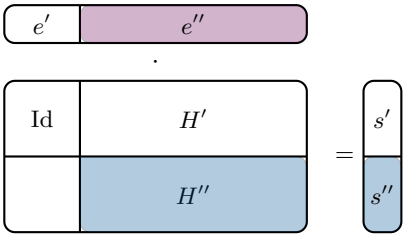
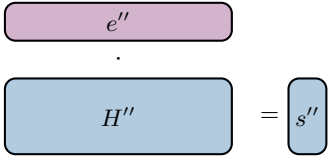


$e' = s' - e''H'^{\top} \in \mathbb{E}$?

- $q = 127, z = 7 \quad \rightarrow \quad \mathbb{E} = \{1, 2, 4, 8, 16, 32, 64\}$ unique sol. $\frac{z^n}{q^{n-k}} < 1$

2. Meet-in-the-Middle



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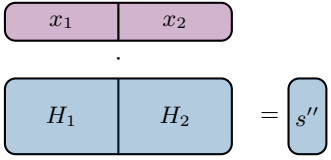

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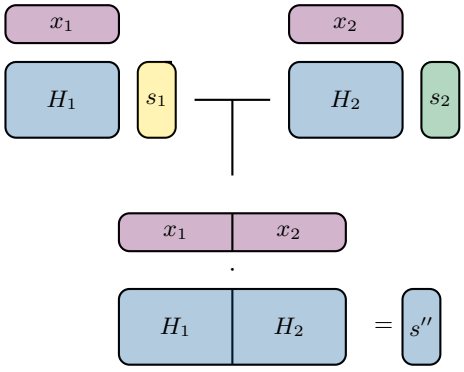

$\rightarrow$ Stern-like solver
for R-SDP, R-SDP(G)

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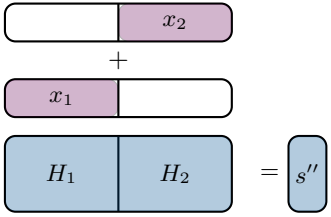
3. Representation technique



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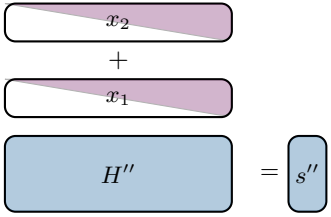

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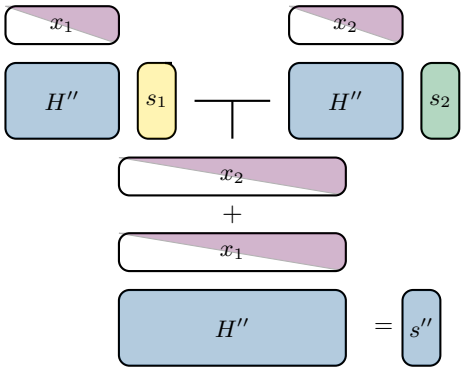

$\rightarrow$ BJMM-like solver for R-SDP

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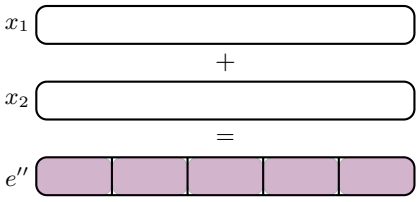

many repr. $(x_1, x_2) : e'' = x_1 + x_2$

small search space for (x_1, x_2)

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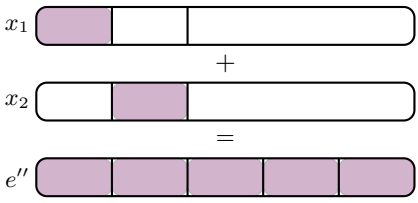

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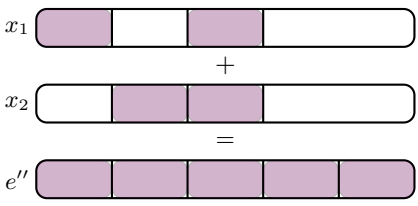

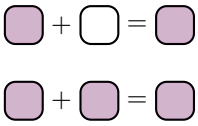

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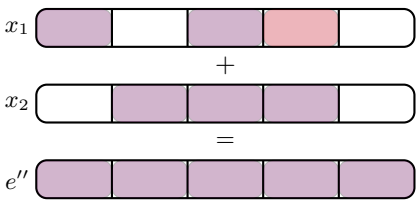
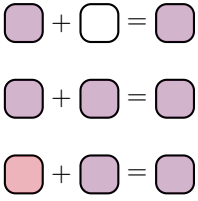
$\mathbb{D} = \mathbb{E} - \mathbb{E} = \{a - b \mid a, b \in \mathbb{E}\}$





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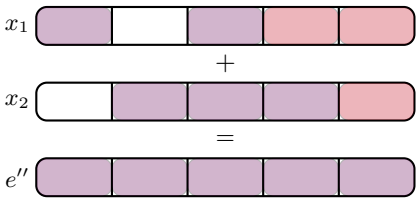

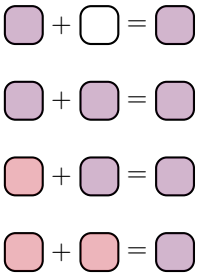

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CROSS - Is this now CROSS?

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weighted challenges

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$$O\left(\min_{t^* \in \{0,\ldots,t\}} \left\{ \frac{1}{P_1(t,t^*,p)} + \frac{1}{P_2(t,t^*,w,p)} \right\}\right),$$

where

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$w > t/2$ $\rightarrow$ small

$w \sim t/2$ $\rightarrow$ fast

$$O\left(\min_{t^* \in \{0,\ldots,t\}} \left\{ \frac{1}{P_1(t,t^*,p)} + \frac{1}{P_2(t,t^*,w,p)} \right\}\right),$$

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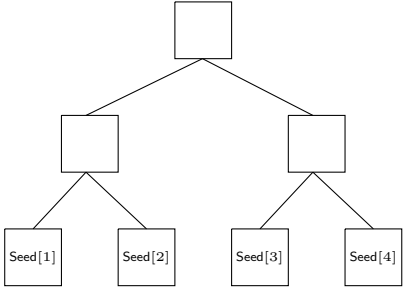
Seed path

round i: $\varphi[i] \leftarrow \mathsf{Seed}[i]$

to send all w requested $\varphi[i]$

$\rightarrow$ can send Path



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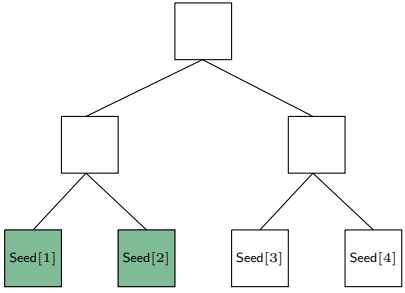

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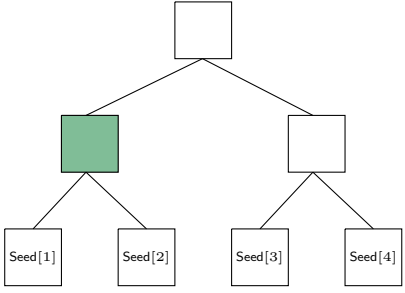
$\rightarrow$ can send Path



not t parallel repetitions $\rightarrow (\beta[1], \ldots, \beta[t]) = \mathsf{Hash}(m, c_1[1], c_2[1], \ldots, c_1[t], c_2[t])$

weighted challenges: $\beta[i] = 1$ w many times

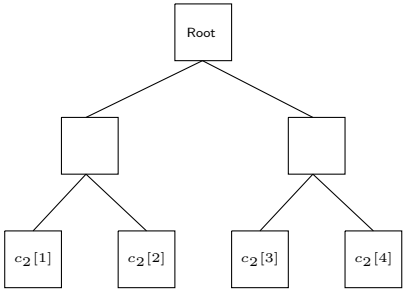
for round i : if challenged with β, put response $r_\beta[i]$ and $c_{\beta'}[i]$ $\rightarrow$ recover both $c_\beta[i], c_{\beta'}[i]$

Merkle proof

include w many $c_2[i]$ in response

use $\mathsf{Hash}(\mathsf{Root})$ instead of $\mathsf{Hash}(c_2[1], \ldots, c_2[t])$

$\rightarrow$ send Proof to recompute Root



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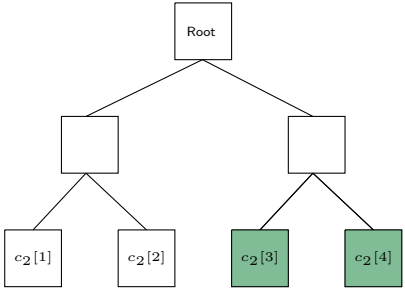

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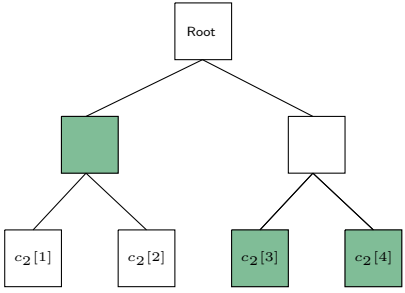
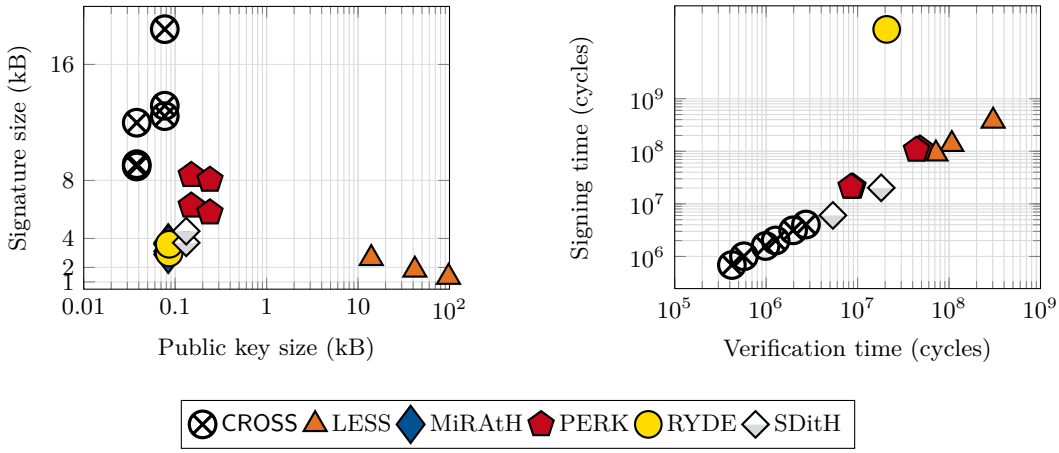


NIST cat. I

Problem	q, z	**Type**	(n, k, m)	(t, w)	|**Sign.**| (kB)	**Sign** (MCycles)	**Verify** (MCycles)
R-SDP	$(127, 7)$	fast	$(127, 76, -)$	$(157, 82)$	18.4	1.01	0.57
		balanced		$(256, 215)$	13.1	2.01	1.27
		short		$(520, 488)$	12.4	4.05	2.73
R-SDP(G)	$(509, 127)$	fast	$(55, 36, 25)$	$(147, 76)$	11.9	0.68	0.42
		balanced		$(256, 220)$	9.1	1.58	0.99
		short		$(512, 484)$	8.9	3.14	1.97

private and public keys < 0.1 kB key gen. < 0.1 MCycle

CROSS - vs. other code-based?



What's next?

- Hardware implementation
- VOLE → smaller signatures

Slides

What's next?

- Hardware implementation
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Slides

Announcement

Talk: Sebastian Bitzer

Title: VOLEitH-based Signatures from Restricted Decoding Problems

Time: 11:50

What's next?

- Hardware implementation
- VOLE → smaller signatures

NIST timeline

2022	Announcement on ramp call
2023	1st round candidates
2024	2nd round candidates
2026	3rd round candidates (finalists?)
2035	RSA, ECDSA disallowed

Slides

What's next?

- Hardware implementation
- VOLE → smaller signatures

Slides

Website

CROSS

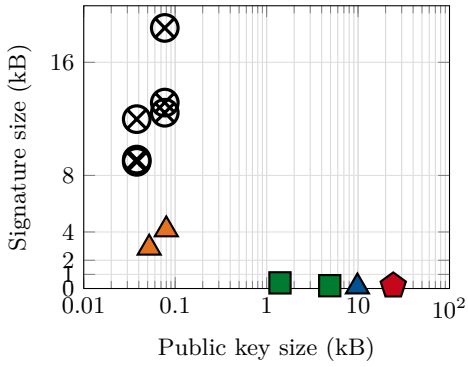
Codes & Restricted Objects Signature Scheme

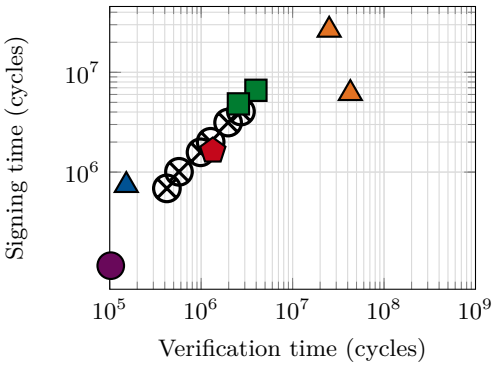
https://cross-crypto.com

Thank you!


TO BE
CONTINUED

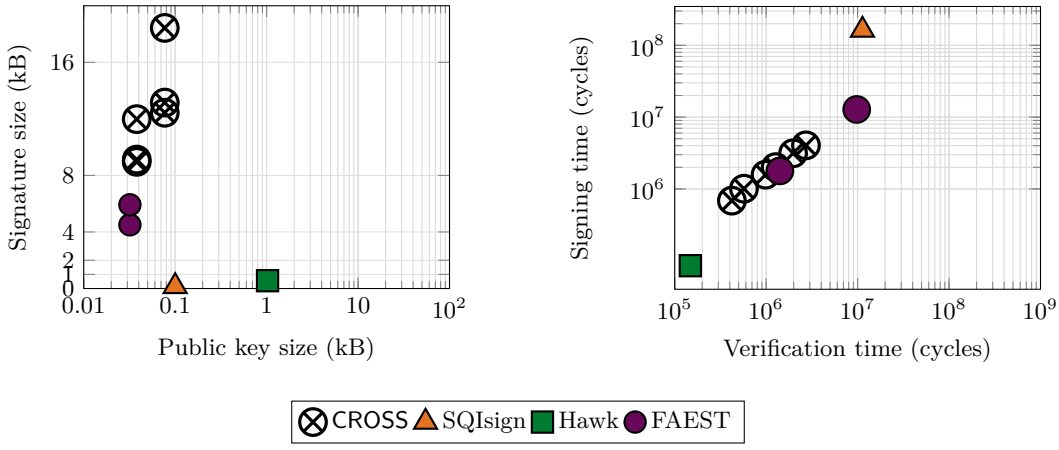
CROSS - vs. Multivariate?





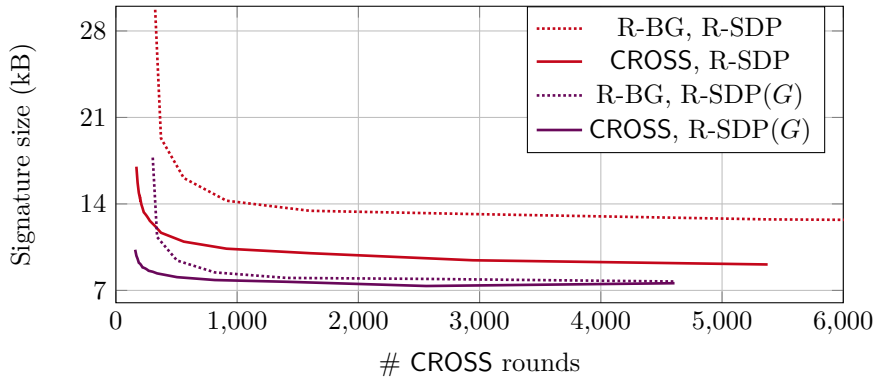
CROSS MQOM MAYO QR-UOV SNOVA UOV

CROSS - vs. rest?



Baldi et al., 2023: R-BG protocol, soundness error $\varepsilon \approx \max\left\{\frac{1}{N}; \frac{1}{q-1}\right\}$

Computational cost: one round of R-BG is $\approx N$ rounds of CROSS



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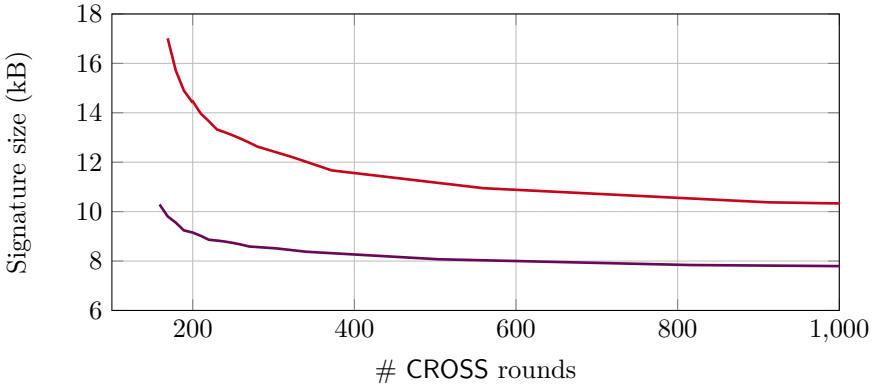

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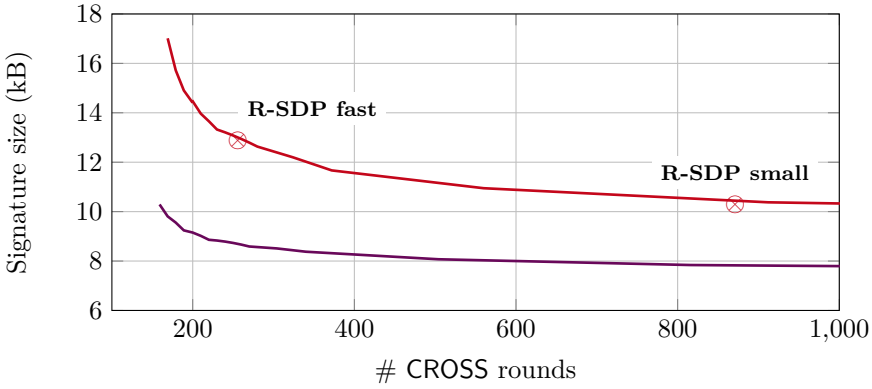

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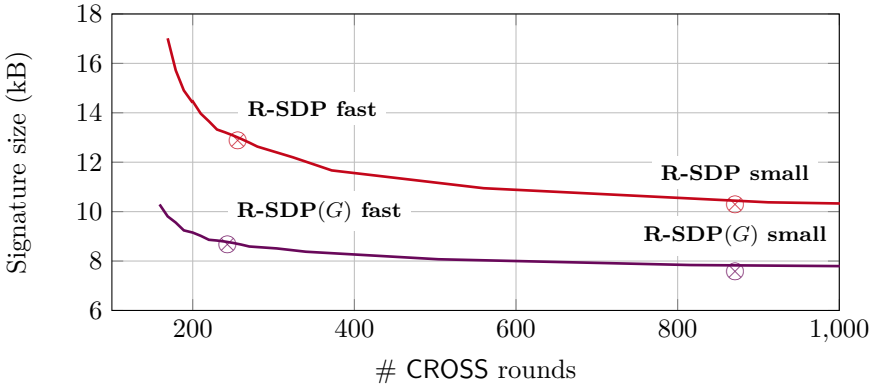


VOLE

Vector Oblivious Linear Transfer

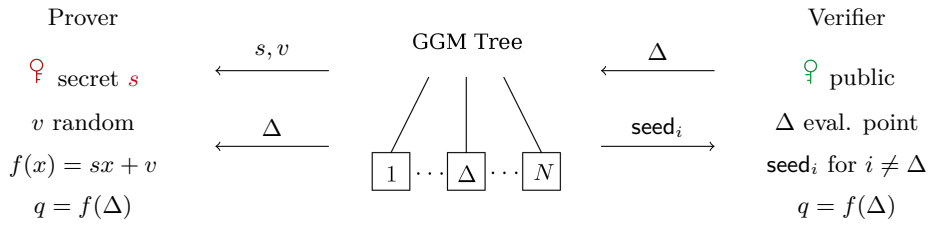
ZK Protocol

Prover	Verifier
secret s	public
v random	Δ eval. point
$f(x) = sx + v$	
	$q = f(\Delta)$

VOLE

Vector Oblivious Linear Transfer

ZK Protocol



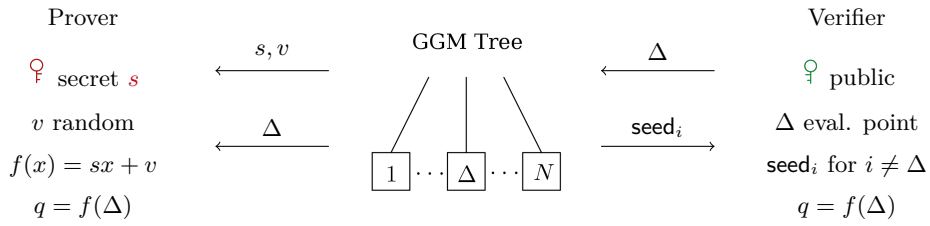
VOLE correlation $q = s\Delta + v = f(\Delta)$

dishonest prover needs to guess Δ before committing to GGM tree: $\mathbb{P} \sim 1/p$

VOLE

Vector Oblivious Linear Transfer

ZK Protocol



MPC

$s = \sum s_i$ MPC $\xleftarrow{\ell}$ $N-1$ views

VOLE

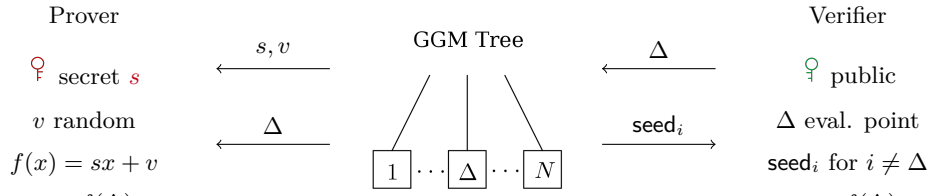
$s = \sum s_i$ GGM $\xleftarrow{\Delta}$ $N-1$ seeds

$v = \sum i s_i$ $q = \sum s_i(\Delta - i) = s\Delta + v$

VOLE

Vector Oblivious Linear Transfer

ZK Protocol



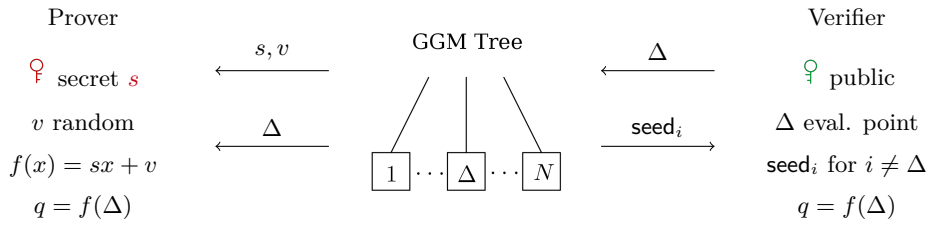
$$f(x) = \sum_{i=0}^{d} f_i x^i,$$
$$s = f_d$$
$$f_1(x), f_2(x)$$

$$f_1(\Delta) + f_2(\Delta) = (f_1 + f_2)(\Delta)$$
$$f_1(\Delta) f_2(\Delta) = (f_1 f_2)(\Delta)$$

VOLE

Vector Oblivious Linear Transfer

ZK Protocol



Disadvantages: slow

Advantages: small sizes